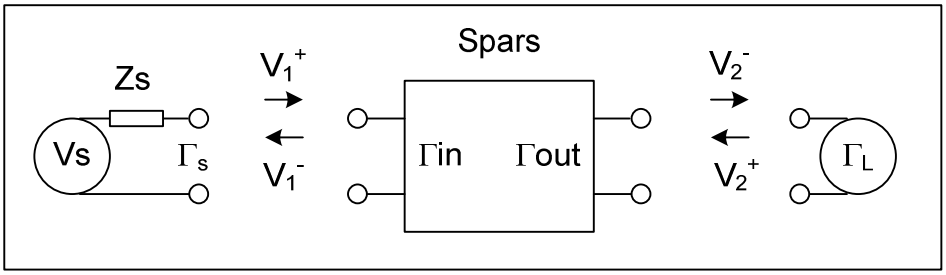
Compilation of S parameter and gain definitions (RW sept06)

$$\Gamma in = \frac{(Zin - Z0)}{(Zin + Z0)} \qquad Zin = Z0\frac{(1+\Gamma in)}{(1-\Gamma in)} \qquad VSWR = \frac{|Zin|}{|Z0|} = \frac{(1+|\Gamma in|)}{(1-|\Gamma in|)} \qquad |\Gamma in| = \frac{VSWR-1}{VSWR+1}$$

$$RL = \text{Return Loss} = 10\log|\Gamma|^2 \qquad ML = \text{Mismatch Loss} = 10\log(1-|\Gamma|^2)$$



$$a1 = \frac{V1^+}{\sqrt{Z0}} \qquad a2 = \frac{V2^+}{\sqrt{Z0}}$$

$$b1 = \frac{V1^-}{\sqrt{Z0}} \qquad b2 = \frac{V2^-}{\sqrt{Z0}}$$

$$V2^- = V1^+S21 + V2^+S22 = V1^+S21 + V2^-\Gamma_L S22 = \frac{V1^+S21}{(1-\Gamma_L S22)}$$

$$V1^- = V1^+S11 + V2^+S12 = V1^+S11 + V2^-\Gamma_L S12 = V1^+S11 + \Gamma_L S12\frac{V1^+S21}{(1-\Gamma_L S22)} = V1^+\left(S11 + \Gamma_L S12\frac{S21}{(1-\Gamma_L S22)}\right)$$

$$\Gamma in = \frac{V1^-}{V1^+} = S11 + \frac{\Gamma_L S12S21}{(1-\Gamma_L S22)} \qquad \text{(pozar.pp241)}$$

$$Pin = \frac{1}{2}|a1|^2 - \frac{1}{2}|b1|^2 = \frac{1}{2}|a1|^2(1-|\Gamma in|^2) = \frac{1}{2}|bs|^2\frac{1-|\Gamma in|^2}{|1-\Gamma s\Gamma in|^2} \qquad \text{(gonzalez.pp188)}$$

$$Pavs = Pin_{[\Gamma in=\Gamma s^*]} = \frac{1}{2}|bs|^2\frac{1}{|1-\Gamma s|^2} \qquad \frac{Pin}{Pavs} = \frac{(1-|\Gamma s|^2)(1-|\Gamma in|^2)}{|1-\Gamma s\Gamma in|^2}$$

$$\frac{Pload}{Pavn} = \frac{(1-|\Gamma_L|^2)(1-|\Gamma out|^2)}{|1-\Gamma_L\Gamma out|^2}$$

G_P= Power gain = delivered to load / delivered to input.
G_T= Transducer gain = delivered to load / available from source.
G_A= Available gain = available from load / available from source.

$$Gp = \frac{Pload}{Pin} = \frac{1}{1-|\Gamma_{in}|^2}|S21|^2\frac{(1-|\Gamma_L|^2)}{|1-\Gamma_L S22|^2} \qquad \Gamma in = S11 + \frac{\Gamma_L S12S21}{(1-\Gamma_L S22)} \qquad Gp_{[\Gamma L\,=\,0]} = \frac{|S21|^2}{1-|S11|^2}$$

$$G_T = \frac{Pload}{Pavs} = \frac{Pload}{Pin}\frac{Pin}{Pavs} = Gp\frac{Pin}{Pavs} = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S\Gamma_{in}|^2}|S21|^2\frac{1-|\Gamma_L|^2}{|1-\Gamma_L S22|^2} = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S S11|^2}|S21|^2\frac{1-|\Gamma_L|^2}{|1-\Gamma_L\Gamma_{OUT}|^2}$$

$$G_A = \frac{Pavn}{Pavs} = \frac{Pload}{Pavs}\frac{Pavn}{Pload} = G_T\frac{Pavn}{Pload} = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S S11|^2}|S21|^2\frac{1}{1-|\Gamma_{OUT}|^2} \qquad \Gamma out = S22 + \frac{\Gamma_S S12S21}{(1-\Gamma_S S11)}$$

$$V1^{+} + V1^{-} = V1 = Vs\frac{Zin}{(Zs + Zin)}$$

$$V1^{+} = Vs\frac{Zin}{(Zs + Zin)} - V1^{-} = Vs\frac{Zin}{(Zs + Zin)} - V1^{+}\Gamma in$$

$$V1^{+}\,(1 + \Gamma in) = Vs\frac{Zin}{(Zs + Zin)}$$

$$Zin = Z0\frac{(1 + \Gamma in)}{(1 - \Gamma in)} \qquad Zs = Z0\frac{(1 + \Gamma s)}{(1 - \Gamma s)}$$

$$V1^{+}\,(1 + \Gamma in) = Vs\left(\frac{Z0\frac{(1 + \Gamma in)}{(1 - \Gamma in)}}{Z0\frac{(1 + \Gamma s)}{(1 - \Gamma s)} + Z0\frac{(1 + \Gamma in)}{(1 - \Gamma in)}}\right)$$

$$V1^{+} = Vs\left(\frac{\frac{1}{(1 - \Gamma in)}}{\frac{(1 + \Gamma s)}{(1 - \Gamma s)} + \frac{(1 + \Gamma in)}{(1 - \Gamma in)}}\right) = Vs\left(\frac{1}{\frac{(1 + \Gamma s)\,(1 - \Gamma in)}{(1 - \Gamma s)} + \frac{(1 + \Gamma in)\,(1 - \Gamma s)}{(1 - \Gamma s)}}\right)$$

$$V1^{+} = Vs\left(\frac{(1 - \Gamma s)}{(1 + \Gamma s)\,(1 - \Gamma in) + (1 + \Gamma in)\,(1 - \Gamma s)}\right) = Vs\left(\frac{(1 - \Gamma s)}{(1 + \Gamma s - \Gamma in - \Gamma s\Gamma in) + (1 + \Gamma in - \Gamma s - \Gamma s\Gamma in)}\right)$$

$$V1^{+} = Vs\left(\frac{(1 - \Gamma s)}{2(1 - \Gamma s\Gamma in)}\right)$$